Ref.TH.3093-CERN

CALCULATION OF THE EIGHTH ORDER ANOMALOUS MAGNETIC MOMENT OF THE ELECTRON *)§)

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A B S T R A C T

We present a very preliminary result of our calculation of the eighth order QED contribution to the anomalous magnetic moment of the electron. Altogether 891 Feynman diagrams contribute to this term. By a method developed earlier we have compressed them into about 100 integrals, which are evaluated using adaptive Monte Carlo integration routines. Our result is $-0.8\ (2.5)(\alpha/\pi)^4$. Combining this with the results of lower orders and using $\alpha^{-1} =$ = 137.035 963 (15) we find

$$a_e^{th} = 1\ 159\ 652\ 460\ (148) \times 10^{-12}.$$

The difference between experiment and theory is now $-251\ (154) \times 10^{-12}$.

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*) Submitted to the Second International Conference on Precision Measurement and Fundamental Constants, National Bureau of Standards, Gaithersburg, MD, 8-12 June, 1981.

§) Supported in part by the U.S. National Science Foundation.

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27 May 1981

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Key words : Anomalous magnetic moment ; fine structure constant ; quantum electrodynamics

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+) 1 April - 31 July, 1981.

The electron belongs to a special class of elementary particles whose interaction is predominantly electromagnetic. Since electromagnetic processes are the easiest to measure and can be done so with the greatest of accuracy, it has provided the main testing ground of quantum electrodynamics (QED). In particular, the magnetic moment anomaly of an electron has been studied both experimentally and theoretically over 30 years with ever-increasing precision. A very substantial progress has been made in the last few years by Dehmelt and his coworkers [1] whose published 1977 value for the magnetic moment anomaly $a_e = (g_e-2)/2$,

$$a_e^{exp} = 1\ 159\ 652\ 410\ (200) \times 10^{-12}, \tag{1}$$

represents a factor of improvement of 20 over the best previous measurement. By 1979 [2] the error went down by a further factor of 5 :

$$a_e^{exp} = 1\ 159\ 652\ 200\ (40) \times 10^{-12}. \tag{2}$$

In perturbation theory of QED, higher order radiative corrections to a_e can be written as a power series in α/π :

$$a_e = C_1(\alpha/\pi) + C_2(\alpha/\pi)^2 + C_3(\alpha/\pi)^3 + C_4(\alpha/\pi)^4 + \ldots \tag{3}$$

Thus far the first three coefficients have been calculated [3] :

$$\begin{aligned} C_1 &= 0.5, \\ C_2 &= -0.328\ 478\ 966\ \ldots, \\ C_3 &= 1.183\ 5\ (61). \end{aligned} \tag{4}$$

If one uses the 1979 value of the fine structure constant α [4]

$$\alpha^{-1} = 137.035\ 963\ (15), \tag{5}$$

the QED prediction (4) gives the value

$$1\ 159\ 652\ 566 \times 10^{-12}. \tag{6}$$

To this one must add contributions from other sources. They include the contributions of the muon loop, the τ meson loop, and the hadronic effect, which are all small :

$$\begin{aligned} a_e\ (\text{muon}) &= 2.8 \times 10^{-12}, \\ a_e\ (\tau\ \text{meson}) &= 0.01 \times 10^{-12}, \\ a_e\ (\text{hadron}) &= 1.6\ (2) \times 10^{-12}. \end{aligned} \tag{7}$$

The effect of the weak interaction (according to the standard Weinberg-Salam model) is also very small [5] :

$$a_e\ (\text{weak}) \simeq 0.05 \times 10^{-12}. \tag{8}$$

Collecting (6), (7) and (8) we obtain the best theoretical prediction available thus far :

$$a_e^{th} = 1\ 159\ 652\ 570 \times 10^{-12}. \tag{9}$$

Comparing this with (2) we see that theory and experiment differ by

$$\Delta a_e = a_e^{exp} - a_e^{th} = -370 \times 10^{-12} \tag{10}$$

which is nearly 10 times as large as the experimental error quoted in (2).

In order to decide whether this discrepancy is significant or not, one must of course examine the errors in the theoretical value (9). The uncertainty in the measurement of α quoted in (5) contributes an error of 127×10^{-12} to (9). The error due to that of C_3 in (4) amounts to 77×10^{-12} which results almost totally from the 15 integrals (out of 72) that have been evaluated only numerically. An additional six integrals (from diagrams containing photon-photon scattering subdiagrams) have recently had their errors reduced by several orders of magnitude by a combined analytical-numerical integration technique [6]. This method, currently being applied to the 15 integrals mentioned above, is expected to reduce the error of C_3 to an insignificant level [7]. The errors in (7) and (8) are completely negligible at this stage. Actually, in view of the fact that

$$(\alpha/\pi)^4 \simeq 29 \times 10^{-12} \tag{11}$$

is almost as large as the experimental error in (2), the most serious theoretical uncertainty arises from the absence of knowledge about C_4 which makes comparison of theory and experiment rather tentative.

On the other hand, if we assume that the difference Δa_e in (10) is genuine and due entirely to the C_4 term, we obtain the "prediction" that

$$C_4 \simeq -13(6), \tag{12}$$

a surprisingly large value. Of course it is entirely possible that the discrepancy (10) is mainly of experimental origin and fades away as measurements improve. Whether this turns out to be the case or not, however, there is no question that we must know the magnitude and sign of C_4 for a meaningful comparison of the theory with the present and forthcoming experiments.

It is for this reason that we decided to calculate the quantity C_4. In the absence of any clever method which enables us to estimate it quickly and reliably, we had no choice but to calculate by brute force the values of all 891 Feynman diagrams that contribute to C_4. However, a substantial simplification has been achieved by a method developed earlier [8] which enables us to combine several integrals into one. In this manner we were able to reduce the number of integrals to be evaluated to slightly over 100.

The 891 Feynman diagrams fall naturally into the following five groups each of which consists of one or more gauge invariant sets.

Group I Second-order vertex diagrams containing vacuum polarization loops of second, fourth and sixth orders. This group consists of 25 diagrams. They are represented by 10 integrals.

Group II Fourth-order vertex diagrams containing vacuum polarization loops of second and fourth orders. This group contains 54 diagrams and is represented by 8 integrals.

Group III Sixth-order vertex diagrams containing a vacuum polarization loop of second order. There are 150 diagrams in this group. The number of independent integrals is 8.

Group IV Vertex diagrams containing a photon-photon scattering subdiagram with further radiative corrections of various kinds. This group consists of 144 diagrams. The number of integrals is 13.

Group V Vertex diagrams containing no vacuum polarization loop. This group is comprised of 518 diagrams and is represented by 47 integrals.

All integrands have been generated by an algebraic programme SCHOONSCHIP. A typical integrand is a rational function consisting of up to 15,000 terms, each term being a product of up to 8 or 9 factors. The integration, over a hypercube of up to 10 dimensions, has been carried out by adaptive Monte Carlo subroutines RIWIAD and VEGAS.

The evaluation of the integrals of the first three groups was completed more than two years ago. The results are [9], [10]

$$C_4^{I} = 0.0766\ (6),$$

$$C_4^{II} = -0.5238\ (10),$$

$$C_4^{III} = 1.419\ (16). \tag{13}$$

The remaining groups are substantially more difficult to evaluate and the results have only recently become available. In view of difficulties encountered in carrying out some of the numerical integrations, the error (90% confidence limit estimated by the integration routines) is still very substantial and should not be fully trusted. Anyway, our very very tentative results are [11]

$$C_4^{IV} = -0.78\ (48)$$

$$C_4^{V} = -1.0\ (2.5) \tag{14}$$

Combining (13) and (14) we obtain

$$C_4 = -0.8\ (2.5) \tag{15}$$

The central value may still fluctuate considerably. The main significance of this result is that we now have finite bounds for C_4, although they may be rather soft. Anyway, it appears that the "prediction" (12) is not borne out by our calculation.

From (4), (5), (7), (8), (15) and Ref. [7] we obtain

$$a_e^{th} = 1\ 159\ 652\ 460\ (148) \times 10^{-12} \qquad (16)$$

This is consistent with the measurement (2) as well as the new measurement of a_e for a positron [12]. Possible causes of remaining discrepancy are underestimate of errors in the measurement of a_e and/or the measurement of α in (5) as well as theoretical errors. The latter includes not only purely computational errors in C_3 and C_4 but also physical assumptions we have implicitly made on the nature of the weak interaction and the (lack of) internal structure of the electron.

An important by-product of our calculation is that it enables us to determine α in a way least dependent on theoretical ambiguities. We give below the value of α determined from the measurement (2) and one quoted in [12] and new calculations of Ref. [7] and this report. For comparison we also list α determined from the muonium hfs [13] and the ac Josephson effect [4]:

$$\begin{aligned} \alpha^{-1}(a_e) &= 137.035\ 993\ (10), \\ \alpha^{-1}(\text{muonium hfs}) &= 137.035\ 989\ (47), \\ \alpha^{-1}(\text{ac Josephson}) &= 137.035\ 963\ (15). \end{aligned} \qquad (17)$$

The error in α^{-1}(momentum hfs) is mostly theoretical and requires a considerable improvement of theory to take advantage of very accurate measurements. Such works are in progress at present [14]. Since it is a pure QED process, comparison of $\alpha^{-1}(a_e)$ and α^{-1}(muonium hfs) is useful for checking the internal consistency of QED. On the other hand, α^{-1}(ac Josephson) is based on a purely solid state physics phenomenon and is presumably not susceptible to higher order QED corrections [15].

As is seen from (17) the agreement of α determined by various means is satisfactory at present. But, irrespective of whether further work establishes the disagreement of different α's or not, it is about time to re-examine the basis of some of the underlying theories very closely. For instance, is the ac Josephson effect really capable of determining $2e/h$ to an accuracy better than 10^{-8} ? What is the upper limit of theoretical error and how does one estimate it ?

Although the new and exciting way of determining α discovered by von Klitzing et al. [16] is not yet comparable in accuracy to that of the ac Josephson effect, a similar question about its theoretical foundation must be asked in order to make it a viable method for high precision determination of α.

Finally I call attention to the very far-fetched, but not completely crazy, possibility that the discrepancy of $\alpha(a_e)$ and other α's arises from the electron not being elementary but having in fact some internal structure which, for instance, may be described as a bound state of subquarks. The present agreement (or disagreement) of theory and experiment of a_e suggests that the mass of such subquarks must be at least of order of 10^6 proton masses [17]. Should this turn out to be the case (which I must say is very unlikely), low energy particle physics and even solid state physics have rôles to play in determining the law of Nature in the domain of ultra-high energy physics.

We should like to thank Brookhaven National Laboratory, National Laboratory for High Energy Physics, Japan, and CERN for generous support of our work.

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